

4/20/18 "Dont cry because its over, smile because it happened."-Dr. Seuss

HW: "Summation Notation" Homework section

AIM: What is Summation Notation?

**SUMMATION (SIGMA) NOTATION**

$\sum_{i=a}^n f(i) = f(a) + f(a+1) + f(a+2) + \dots + f(n)$

where  $i$  is called the **index variable**, which starts at a value of  $a$ , ends at a value of  $n$ , and moves by unit increments (increase by 1 each time).

Last  
inputStarting  
inputformula  
we plug inputs into**Exercise #1:** Evaluate each of the following sums.

(a)  $\sum_{i=3}^5 2i$   $2(3) + 2(4) + 2(5) =$   
 $6 + 8 + 10$   
 Start with 3  
 end with 5 24

(b)  $\sum_{k=-1}^3 k^2 = (-1)^2 + (0)^2 + (1)^2 + (2)^2 + (3)^2$   
 $1 + 0 + 1 + 4 + 9$   
15

(c)  $\sum_{j=-2}^2 2^j = 2^{-2} + 2^{-1} + 2^0 + 2^1 + 2^2$   
 $\frac{1}{2^2} \quad \frac{1}{2}$   
 $\frac{1}{4} + \frac{1}{2} + 1 + 2 + 4 = \frac{31}{4} = 7.75$

$\sum_{x=1}^{x=3} (3n)^3$

**Exercise #2:** Which of represents the value of  $\sum_{i=1}^4 \frac{1}{i}$ ?

(1)  $\frac{1}{10}$

(3)  $\frac{25}{12}$

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$$

(2)  $\frac{9}{4}$

(4)  $\frac{31}{24}$

$$\frac{12}{12} + \frac{6}{12} + \frac{4}{12} + \frac{3}{12}$$

**Exercise #3:** Consider the sequence defined recursively by  $a_n = a_{n-1} + 2a_{n-2}$  and  $a_1 = 0$  and  $a_2 = 1$ . Find the value of  $\sum_{i=4}^7 a_i$

**Exercise #4:** Express each sum using sigma notation. Use  $i$  as your index variable. First, consider any patterns you notice amongst the terms involved in the sum. Then, work to put these patterns into a formula and sum.

(a)  $9 + 16 + 25 + \dots + 100$

$\downarrow \quad \downarrow \quad \downarrow \quad \dots \quad \downarrow$   
 $3^2 \quad 4^2 \quad 5^2 \quad \dots \quad 10^2$

⊗ look for part to stay the same AND some part to change by 1 every time.

(b)  $-6 + -3 + 0 + 3 + \dots + 15$   
increase by 3

$$\sum_{x=0}^7 (-6 + 3(x))$$

$$\begin{aligned} -6 + 3(x) &= 15 \\ 3x &= 21 \\ x &= 7 \end{aligned}$$

$$\sum_{x=1}^6 (-9 + 3(x))$$

(c)  $\frac{1}{25} + \frac{1}{5} + 1 + 5 + \dots + 625$

**Exercise #5:** Which of the following represents the sum  $3 + 6 + 12 + 24 + 48$ ?

~~(1)  $\sum_{i=1}^5 3^i$~~

~~(3)  $\sum_{i=0}^4 6^{i-1}$~~

(2)  $\sum_{i=0}^4 3(2)^i$

~~(4)  $\sum_{i=3}^{48} i$~~

**Exercise #6:** Some sums are more interesting than others. Determine the value of  $\sum_{i=1}^{99} \left( \frac{1}{i} - \frac{1}{i+1} \right)$ . Show your reasoning. This is known as a **telescoping series (or sum)**.